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RESOLUTION OF THE PERSECUTOR — PREY SYSTEM ON THE UNIT HYPERCUBE FOR THE EXPONENTIAL ANNIHILATION PROBABILITY

There has been solved the persecutor — prey system, stated as the antagonistic game, being defined on the unit hypercube of the space $\mathbb{R}^4$ with the determined half-open interval positive parameter. For solving the game with the greater values of this parameter there should be applied the constructed program module, returning the approximate solution, though its accuracy may be anyhow increased on protracting the resolution process.

Решено систему преследователь — добыча, сформулированную в виде антагонистической игры, которая задаётся на единичном гиперкубе пространства $\mathbb{R}^4$ с определённым на полуоткрытом интервале положительным параметром. Для решения игры с большими значениями этого параметра должен быть применён построенный программный модуль, что возвращает приближённое решение, хотя его точность может быть как угодно увеличена при продлении процесса решения.

Problem exposure and paper destination. The antagonistic gaming is a fascinating apparatus for constructing the mathematical models within the conflict processes. One of the most attractive conflict processes is the persecutor — prey system evolution, considered on different aspects in the origins [1 — 3]. May there be the persecutor, shooting the acquired target, maneuvering within some rectangular area. For the target total annihilation the persecutor shoots this rectangular area, though making it without the exact coordinates of the targeted prey. The task of the persecutor is to annihilate the prey, and the task of the prey is to avoid the annihilation. If the rectangular area is conditionally squeezed to the unit square $[0;\,1]\times[0;\,1]$ then the described conflict process may be claimed the antagonistic game, where the persecutor has the unit square

$$U_{\mathbf{X}} = X_1 \times X_2 = [0;\,1]\times[0;\,1] \quad (1)$$

of its pure strategies, and the prey has the unit square

$$U_{\mathbf{Y}} = Y_1 \times Y_2 = [0;\,1]\times[0;\,1] \quad (2)$$

of its pure strategies. Consequently, a persecutor pure strategy is the plane square point

$$\mathbf{X} = [x_1 \quad x_2] \in U_{\mathbf{X}} \subset \mathbb{R}^2, \quad (3)$$

and a prey pure strategy is the plane square point

$$\mathbf{Y} = [y_1 \quad y_2] \in U_{\mathbf{Y}} \subset \mathbb{R}^2. \quad (4)$$

It is known, that the probability of the prey annihilation is exponential [1, p. 63], and as the persecutor selects the points (3) from the area (1) independently from the prey, selecting the points (4) from the area (2), then this probability is

$$P(\mathbf{X},\,\mathbf{Y}) = P(x_1,\,y_1,\,x_2,\,y_2) = \left(\exp\left[-\alpha(x_1-y_1)^2\right]\right)\cdot\left(\exp\left[-\alpha(x_2-y_2)^2\right]\right) = $$
$$= \exp\left[-\alpha(x_1-y_1)^2 - \alpha(x_2-y_2)^2\right] \quad (5)$$

by the parameter $\alpha > 0$. The lesser this parameter the persecutor equipment is more perfect. And this paper destination is to solve the claimed antagonistic game with the kernel (5), defined on the unit hypercube

$$H_P = U_{\mathbf{X}} \times U_{\mathbf{Y}} = X_1 \times X_2 \times Y_1 \times Y_2 = \prod_{k=1}^{4}[0;\,1] \quad (6)$$

by the first player, having the pure strategies set (1), and the second player, having the pure strategies set (2). The solution will allow to optimize the behavior as in the military anti-aircraft defense, as well as in the civil competitive conformations, moving and acting on the plane surface.

Unit hypercube system solution. Primarily the stated game should be verified for its concavity and convexity, what may assist in solving the game analytically [2, 4, 5]. The first partial derivative of the hypersurface (5) by the variable x_1 is

$$\frac{\partial P(x_1, y_1, x_2, y_2)}{\partial x_1} = \frac{\partial}{\partial x_1}\left(\exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right]\right) =$$
$$= -2\alpha(x_1 - y_1)\exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right] \quad (7)$$

and its second partial derivative by the variable x_1 is

$$\frac{\partial^2 P(x_1, y_1, x_2, y_2)}{\partial x_1 \partial x_1} = \frac{\partial}{\partial x_1}\left(-2\alpha(x_1 - y_1)\exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right]\right) =$$
$$= -2\alpha \exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right] + 4\alpha^2 (x_1 - y_1)^2 \exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right] =$$
$$= 2\alpha\left(2\alpha(x_1 - y_1)^2 - 1\right)\exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right]. \quad (8)$$

The first partial derivative of the hypersurface (5) by the variable x_2 is

$$\frac{\partial P(x_1, y_1, x_2, y_2)}{\partial x_2} = \frac{\partial}{\partial x_2}\left(\exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right]\right) =$$
$$= -2\alpha(x_2 - y_2)\exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right] \quad (9)$$

and its second partial derivative by the variable x_2 is

$$\frac{\partial^2 P(x_1, y_1, x_2, y_2)}{\partial x_2 \partial x_2} = \frac{\partial}{\partial x_2}\left(-2\alpha(x_2 - y_2)\exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right]\right) =$$
$$= -2\alpha \exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right] + 4\alpha^2 (x_2 - y_2)^2 \exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right] =$$
$$= 2\alpha\left(2\alpha(x_2 - y_2)^2 - 1\right)\exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right]. \quad (10)$$

The first partial derivative of the hypersurface (5) by the variable y_1 is

$$\frac{\partial P(x_1, y_1, x_2, y_2)}{\partial y_1} = \frac{\partial}{\partial y_1}\left(\exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right]\right) =$$
$$= 2\alpha(x_1 - y_1)\exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right] \quad (11)$$

and its second partial derivative by the variable y_1 is

$$\frac{\partial^2 P(x_1, y_1, x_2, y_2)}{\partial y_1 \partial y_1} = \frac{\partial}{\partial y_1}\left(2\alpha(x_1 - y_1)\exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right]\right) =$$
$$= -2\alpha \exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right] + 4\alpha^2 (x_1 - y_1)^2 \exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right] =$$
$$= 2\alpha\left(2\alpha(x_1 - y_1)^2 - 1\right)\exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right] = \frac{\partial^2 P(x_1, y_1, x_2, y_2)}{\partial x_1 \partial x_1}. \quad (12)$$

The first partial derivative of the hypersurface (5) by the variable y_2 is

$$\frac{\partial P(x_1, y_1, x_2, y_2)}{\partial y_2} = \frac{\partial}{\partial y_2}\left(\exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right]\right) =$$

$$= 2\alpha\left(x_2 - y_2\right)\exp\left[-\alpha\left(x_1 - y_1\right)^2 - \alpha\left(x_2 - y_2\right)^2\right] \tag{13}$$

and its second partial derivative by the variable y_2 is

$$\frac{\partial^2 P\left(x_1,\, y_1,\, x_2,\, y_2\right)}{\partial y_2 \partial y_2} = \frac{\partial}{\partial y_2}\left(2\alpha\left(x_2 - y_2\right)\exp\left[-\alpha\left(x_1 - y_1\right)^2 - \alpha\left(x_2 - y_2\right)^2\right]\right) =$$
$$= -2\alpha\exp\left[-\alpha\left(x_1 - y_1\right)^2 - \alpha\left(x_2 - y_2\right)^2\right] + 4\alpha^2\left(x_2 - y_2\right)^2\exp\left[-\alpha\left(x_1 - y_1\right)^2 - \alpha\left(x_2 - y_2\right)^2\right] =$$
$$= 2\alpha\left(2\alpha\left(x_2 - y_2\right)^2 - 1\right)\exp\left[-\alpha\left(x_1 - y_1\right)^2 - \alpha\left(x_2 - y_2\right)^2\right] = \frac{\partial^2 P\left(x_1,\, y_1,\, x_2,\, y_2\right)}{\partial x_2 \partial x_2}. \tag{14}$$

The hypersurface strict concavity conditions

$$\frac{\partial^2 P\left(x_1,\, y_1,\, x_2,\, y_2\right)}{\partial x_1 \partial x_1} < 0 \tag{15}$$

and

$$\frac{\partial^2 P\left(x_1,\, y_1,\, x_2,\, y_2\right)}{\partial x_2 \partial x_2} < 0 \tag{16}$$

must be simultaneously held true for any points (3) and (4). As there is the double inequality

$$0 < \exp\left[-\alpha\left(x_1 - y_1\right)^2 - \alpha\left(x_2 - y_2\right)^2\right] \leqslant 1, \tag{17}$$

then the inequality

$$2\alpha\left(2\alpha\left(x_1 - y_1\right)^2 - 1\right)\exp\left[-\alpha\left(x_1 - y_1\right)^2 - \alpha\left(x_2 - y_2\right)^2\right] < 0 \tag{18}$$

is identical to the inequality

$$2\alpha\left(x_1 - y_1\right)^2 - 1 < 0. \tag{19}$$

Analogously with the double inequality (17) the inequality

$$2\alpha\left(2\alpha\left(x_2 - y_2\right)^2 - 1\right)\exp\left[-\alpha\left(x_1 - y_1\right)^2 - \alpha\left(x_2 - y_2\right)^2\right] < 0 \tag{20}$$

turns into the inequality

$$2\alpha\left(x_2 - y_2\right)^2 - 1 < 0, \tag{21}$$

whence, minding the obvious factors $\left(x_1 - y_1\right)^2 \in \left[0;\, 1\right]$ and $\left(x_2 - y_2\right)^2 \in \left[0;\, 1\right]$, the coming out from the inequalities (19) and (21) parameter $\alpha \in \left(0;\, \frac{1}{2}\right)$ determines the strict concavity of the antagonistic game with the kernel (5), defined on the unit hypercube (6). It is clear that by the parameter $\alpha = \frac{1}{2}$ the stated game becomes nonstrictly concave.

Going further, as the hypersurface strict convexity conditions

$$\frac{\partial^2 P\left(x_1,\, y_1,\, x_2,\, y_2\right)}{\partial y_1 \partial y_1} > 0 \tag{22}$$

and

$$\frac{\partial^2 P(x_1, y_1, x_2, y_2)}{\partial y_2 \partial y_2} > 0 \tag{23}$$

must be simultaneously held true for any points (3) and (4), then, having used the double inequality (17) up with the needful inequality

$$2\alpha\left(2\alpha(x_1 - y_1)^2 - 1\right)\exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right] > 0, \tag{24}$$

the caught out strict convexity conditions

$$2\alpha(x_1 - y_1)^2 - 1 > 0 \tag{25}$$

and

$$2\alpha(x_2 - y_2)^2 - 1 > 0 \tag{26}$$

are impracticable for any $\alpha > 0$ as they fail yet for $x_1 = y_1$ or $x_2 = y_2$.

Consequently, the game with the parameter $\alpha > \frac{1}{2}$ will be solved numerically, and now, having put the parameter $\alpha \in \left(0; \frac{1}{2}\right]$ for the game nonstrict concavity, the solution lies in the following statements. Firstly there should be determined the minimum of the hypersurface (5) on the unit square (2) by the variable $\mathbf{Y}$. The minimum of the hypersurface (5) as the surface function of the variables y_1 and y_2 on the unit square (2) is

$$\begin{gathered} \min_{\mathbf{Y} \in U_{\mathbf{Y}}} P(\mathbf{X}, \mathbf{Y}) = \min_{y_1 \in [0;\, 1]} \min_{y_2 \in [0;\, 1]} P(x_1, y_1, x_2, y_2) = \\ = \min_{y_1 \in [0;\, 1]} \min_{y_2 \in [0;\, 1]} \exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right] = \min_{y_1 \in [0;\, 1]} P(x_1, y_1, x_2, 1) = \\ = \min_{y_1 \in [0;\, 1]} \exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - 1)^2\right] \;\; \forall\, x_2 \in \left[0; \frac{1}{2}\right] \end{gathered} \tag{27}$$

and

$$\begin{gathered} \min_{\mathbf{Y} \in U_{\mathbf{Y}}} P(\mathbf{X}, \mathbf{Y}) = \min_{y_1 \in [0;\, 1]} \min_{y_2 \in [0;\, 1]} P(x_1, y_1, x_2, y_2) = \\ = \min_{y_1 \in [0;\, 1]} \min_{y_2 \in [0;\, 1]} \exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - y_2)^2\right] = \min_{y_1 \in [0;\, 1]} P(x_1, y_1, x_2, 0) = \\ = \min_{y_1 \in [0;\, 1]} \exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2)^2\right] \;\; \forall\, x_2 \in \left[\frac{1}{2}; 1\right], \end{gathered} \tag{28}$$

whereupon

$$\begin{gathered} \min_{\mathbf{Y} \in U_{\mathbf{Y}}} P(\mathbf{X}, \mathbf{Y}) = \min_{y_1 \in [0;\, 1]} \min_{y_2 \in [0;\, 1]} P(x_1, y_1, x_2, y_2) = \min_{y_1 \in [0;\, 1]} P(x_1, y_1, x_2, 1) = \\ = \min_{y_1 \in [0;\, 1]} \exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - 1)^2\right] = P(x_1, 1, x_2, 1) = \\ = \exp\left[-\alpha(x_1 - 1)^2 - \alpha(x_2 - 1)^2\right] \;\; \forall\, x_1 \in \left[0; \frac{1}{2}\right] \text{ and } \forall\, x_2 \in \left[0; \frac{1}{2}\right], \end{gathered} \tag{29}$$

$$\begin{gathered} \min_{\mathbf{Y} \in U_{\mathbf{Y}}} P(\mathbf{X}, \mathbf{Y}) = \min_{y_1 \in [0;\, 1]} \min_{y_2 \in [0;\, 1]} P(x_1, y_1, x_2, y_2) = \min_{y_1 \in [0;\, 1]} P(x_1, y_1, x_2, 1) = \\ = \min_{y_1 \in [0;\, 1]} \exp\left[-\alpha(x_1 - y_1)^2 - \alpha(x_2 - 1)^2\right] = P(x_1, 0, x_2, 1) = \\ = \exp\left[-\alpha(x_1)^2 - \alpha(x_2 - 1)^2\right] \;\; \forall\, x_1 \in \left[\frac{1}{2}; 1\right] \text{ and } \forall\, x_2 \in \left[0; \frac{1}{2}\right], \end{gathered} \tag{30}$$

$$\min_{\mathbf{Y} \in U_{\mathbf{Y}}} P(\mathbf{X}, \mathbf{Y}) = \min_{y_1 \in [0;\, 1]} \min_{y_2 \in [0;\, 1]} P(x_1, y_1, x_2, y_2) = \min_{y_1 \in [0;\, 1]} P(x_1, y_1, x_2, 0) =$$

$$= \min_{y_1 \in [0;\,1]} \exp\left[-\alpha\left(x_1 - y_1\right)^2 - \alpha\left(x_2\right)^2\right] = P\left(x_1,\, 1,\, x_2,\, 0\right) =$$

$$= \exp\left[-\alpha\left(x_1 - 1\right)^2 - \alpha\left(x_2\right)^2\right] \;\; \forall\; x_1 \in \left[0;\, \frac{1}{2}\right] \text{ and } \forall\; x_2 \in \left[\frac{1}{2};\, 1\right], \tag{31}$$

$$\min_{\mathbf{Y} \in U_{\mathbf{Y}}} P\left(\mathbf{X},\, \mathbf{Y}\right) = \min_{y_1 \in [0;\,1]} \min_{y_2 \in [0;\,1]} P\left(x_1,\, y_1,\, x_2,\, y_2\right) = \min_{y_1 \in [0;\,1]} P\left(x_1,\, y_1,\, x_2,\, 0\right) =$$

$$= \min_{y_1 \in [0;\,1]} \exp\left[-\alpha\left(x_1 - y_1\right)^2 - \alpha\left(x_2\right)^2\right] = P\left(x_1,\, 0,\, x_2,\, 0\right) =$$

$$= \exp\left[-\alpha\left(x_1\right)^2 - \alpha\left(x_2\right)^2\right] \;\; \forall\; x_1 \in \left[\frac{1}{2};\, 1\right] \text{ and } \forall\; x_2 \in \left[\frac{1}{2};\, 1\right]. \tag{32}$$

Then the optimal game value

$$\theta_{\text{opt}} = \max_{\mathbf{X} \in U_{\mathbf{X}}} \min_{\mathbf{Y} \in U_{\mathbf{Y}}} P\left(\mathbf{X},\, \mathbf{Y}\right) = \max_{x_1 \in [0;\,1]} \max_{x_2 \in [0;\,1]} \min_{y_1 \in [0;\,1]} \min_{y_2 \in [0;\,1]} P\left(x_1,\, y_1,\, x_2,\, y_2\right) =$$

$$= \max\left\{ \max_{x_1 \in \left[0;\, \frac{1}{2}\right]} \max_{x_2 \in \left[0;\, \frac{1}{2}\right]} P\left(x_1,\, 1,\, x_2,\, 1\right),\; \max_{x_1 \in \left[\frac{1}{2};\, 1\right]} \max_{x_2 \in \left[0;\, \frac{1}{2}\right]} P\left(x_1,\, 0,\, x_2,\, 1\right),\right.$$

$$\left. \max_{x_1 \in \left[0;\, \frac{1}{2}\right]} \max_{x_2 \in \left[\frac{1}{2};\, 1\right]} P\left(x_1,\, 1,\, x_2,\, 0\right),\; \max_{x_1 \in \left[\frac{1}{2};\, 1\right]} \max_{x_2 \in \left[\frac{1}{2};\, 1\right]} P\left(x_1,\, 0,\, x_2,\, 0\right) \right\} =$$

$$= \max\left\{ \max_{x_1 \in \left[0;\, \frac{1}{2}\right]} P\left(x_1,\, 1,\, \frac{1}{2},\, 1\right),\; \max_{x_1 \in \left[\frac{1}{2};\, 1\right]} P\left(x_1,\, 0,\, \frac{1}{2},\, 1\right),\right.$$

$$\left. \max_{x_1 \in \left[0;\, \frac{1}{2}\right]} P\left(x_1,\, 1,\, \frac{1}{2},\, 0\right),\; \max_{x_1 \in \left[\frac{1}{2};\, 1\right]} P\left(x_1,\, 0,\, \frac{1}{2},\, 0\right) \right\} =$$

$$= \max\left\{ P\left(\frac{1}{2},\, 1,\, \frac{1}{2},\, 1\right),\, P\left(\frac{1}{2},\, 0,\, \frac{1}{2},\, 1\right),\, P\left(\frac{1}{2},\, 1,\, \frac{1}{2},\, 0\right),\, P\left(\frac{1}{2},\, 0,\, \frac{1}{2},\, 0\right) \right\} =$$

$$= \max\left\{ \exp\left(-\frac{\alpha}{2}\right),\, \exp\left(-\frac{\alpha}{2}\right),\, \exp\left(-\frac{\alpha}{2}\right),\, \exp\left(-\frac{\alpha}{2}\right) \right\} = \exp\left(-\frac{\alpha}{2}\right) \tag{33}$$

is reached [2, p. 86] on the single optimal pure strategy of the persecutor

$$\mathbf{X}_{\text{opt}} = \left[x_1^{\langle \text{opt} \rangle} \quad x_2^{\langle \text{opt} \rangle} \right] = \left[\frac{1}{2} \quad \frac{1}{2} \right]. \tag{34}$$

And this optimal pure strategy is the generally single optimal strategy of the persecutor.

The equilibrant pure strategies of the prey are the roots [6, p. 121 — 125, p. 139] of the standard equation

$$\theta_{\text{opt}} = \exp\left(-\frac{\alpha}{2}\right) = P\left(\mathbf{X}_{\text{opt}},\, \mathbf{Y}\right) = P\left(x_1^{\langle \text{opt} \rangle},\, y_1,\, x_2^{\langle \text{opt} \rangle},\, y_2\right) = P\left(\frac{1}{2},\, y_1,\, \frac{1}{2},\, y_2\right) =$$

$$= \exp\left[-\alpha\left(\frac{1}{2} - y_1\right)^2 - \alpha\left(\frac{1}{2} - y_2\right)^2\right]. \tag{35}$$

Exponentiating both parts of the equation (35) and transferring from the equality

$$\frac{\alpha}{2} = \alpha\left(\frac{1}{2} - y_1\right)^2 + \alpha\left(\frac{1}{2} - y_2\right)^2 \tag{36}$$

to the equality

$$y_1\left(y_1 - 1\right) + y_2\left(y_2 - 1\right) = 0\,, \tag{37}$$

it is seen from the equation (37), that the roots of the equation (35) are

$$\mathbf{Y}_1 = \left[y_1^{\langle 1 \rangle} \quad y_2^{\langle 1 \rangle} \right] = \left[0 \quad 0 \right], \tag{38}$$

$$\mathbf{Y}_2 = \left[y_1^{\langle 2 \rangle} \quad y_2^{\langle 2 \rangle} \right] = \left[1 \quad 0 \right] = \left[y_1^{\langle 2 \rangle} \quad y_2^{\langle 1 \rangle} \right], \tag{39}$$

$$\mathbf{Y}_3 = \left[y_1^{\langle 3 \rangle} \quad y_2^{\langle 3 \rangle} \right] = \left[0 \quad 1 \right] = \left[y_1^{\langle 1 \rangle} \quad y_2^{\langle 3 \rangle} \right], \tag{40}$$

$$\mathbf{Y}_4 = \left[y_1^{\langle 4 \rangle} \quad y_2^{\langle 4 \rangle} \right] = \left[1 \quad 1 \right] = \left[y_1^{\langle 2 \rangle} \quad y_2^{\langle 3 \rangle} \right]. \tag{41}$$

Then the optimal strategy of the prey is represented as the probabilities vector

$$\mathbf{Q}_{\text{opt}} = \left[q_{\text{opt}}\left(\mathbf{Y}_1\right) \quad q_{\text{opt}}\left(\mathbf{Y}_2\right) \quad q_{\text{opt}}\left(\mathbf{Y}_3\right) \quad q_{\text{opt}}\left(\mathbf{Y}_4\right) \right] =$$
$$= \left[q_{\text{opt}}\left(y_1^{\langle 1 \rangle},\, y_2^{\langle 1 \rangle}\right) \quad q_{\text{opt}}\left(y_1^{\langle 2 \rangle},\, y_2^{\langle 1 \rangle}\right) \quad q_{\text{opt}}\left(y_1^{\langle 1 \rangle},\, y_2^{\langle 3 \rangle}\right) \quad q_{\text{opt}}\left(y_1^{\langle 2 \rangle},\, y_2^{\langle 3 \rangle}\right) \right] =$$
$$= \left[q_{\text{opt}}\left(0,\, 0\right) \quad q_{\text{opt}}\left(1,\, 0\right) \quad q_{\text{opt}}\left(0,\, 1\right) \quad q_{\text{opt}}\left(1,\, 1\right) \right] \in \mathbb{R}^4 \tag{42}$$

by the sum

$$\sum_{j=1}^{4} q_{\text{opt}}\left(\mathbf{Y}_j\right) = q_{\text{opt}}\left(0,\, 0\right) + q_{\text{opt}}\left(1,\, 0\right) + q_{\text{opt}}\left(0,\, 1\right) + q_{\text{opt}}\left(1,\, 1\right) = 1\,, \tag{43}$$

where the left inequality

$$\sum_{j=1}^{4} q_{\text{opt}}\left(\mathbf{Y}_j\right) P\left(\mathbf{X},\, \mathbf{Y}_j\right) \leqslant \theta_{\text{opt}} = \exp\left(-\frac{\alpha}{2}\right) \tag{44}$$

should be satisfied for any points (3) and (4) in the equilibrium point double inequality

$$\sum_{j=1}^{4} q_{\text{opt}}\left(\mathbf{Y}_j\right) P\left(\mathbf{X},\, \mathbf{Y}_j\right) \leqslant \theta_{\text{opt}} = \exp\left(-\frac{\alpha}{2}\right) \leqslant \int_0^1 \int_0^1 \mu\left(\mathbf{Y}\right) P\left(\mathbf{X}_{\text{opt}},\, \mathbf{Y}\right) dy_1 dy_2 \tag{45}$$

for any probability distribution density function $\mu\left(\mathbf{Y}\right)$ with the normality condition

$$\int_0^1 \int_0^1 \mu\left(\mathbf{Y}\right) dy_1 dy_2 = \int_0^1 \int_0^1 \mu\left(y_1,\, y_2\right) dy_1 dy_2 = 1\,. \tag{46}$$

Accomplish the needful transformations over the inequality (44) for determining the vector (42), having minded the condition (43):

$$\sum_{j=1}^{4} q_{\text{opt}}\left(\mathbf{Y}_j\right) P\left(\mathbf{X},\, \mathbf{Y}_j\right) = q_{\text{opt}}\left(0,\, 0\right) \exp\left[-\alpha\left(x_1\right)^2 - \alpha\left(x_2\right)^2 \right] +$$
$$+ q_{\text{opt}}\left(1,\, 0\right) \exp\left[-\alpha\left(x_1 - 1\right)^2 - \alpha\left(x_2\right)^2 \right] + q_{\text{opt}}\left(0,\, 1\right) \exp\left[-\alpha\left(x_1\right)^2 - \alpha\left(x_2 - 1\right)^2 \right] +$$
$$+ q_{\text{opt}}\left(1,\, 1\right) \exp\left[-\alpha\left(x_1 - 1\right)^2 - \alpha\left(x_2 - 1\right)^2 \right] \leqslant \exp\left(-\frac{\alpha}{2}\right) \tag{47}$$

and

$$\Big(q_{\text{opt}}\left(0,\, 0\right) + q_{\text{opt}}\left(1,\, 0\right) \exp\left[-\alpha\left(1 - 2x_1\right) \right] + q_{\text{opt}}\left(0,\, 1\right) \exp\left[-\alpha\left(1 - 2x_2\right) \right] +$$
$$+ q_{\text{opt}}\left(1,\, 1\right) \exp\left[-\alpha\left(1 - 2x_1\right) - \alpha\left(1 - 2x_2\right) \right] \Big) \exp\left(-\alpha\left[\left(x_1\right)^2 + \left(x_2\right)^2 \right] \right) \leqslant \exp\left(-\frac{\alpha}{2}\right), \tag{48}$$

whence

$$q_{\text{opt}}(0,\,0)+q_{\text{opt}}(1,\,0)\exp\left[-\alpha\left(1-2x_1\right)\right]+q_{\text{opt}}(0,\,1)\exp\left[-\alpha\left(1-2x_2\right)\right]+$$
$$+q_{\text{opt}}(1,\,1)\exp\left[-\alpha\left(1-2x_1\right)-\alpha\left(1-2x_2\right)\right]\leqslant\exp\left(\alpha\left[\left(x_1\right)^2+\left(x_2\right)^2\right]-\frac{\alpha}{2}\right) \tag{49}$$

as there is the inequality

$$0<\exp\left(-2\alpha\right)\leqslant\exp\left(-\alpha\left[\left(x_1\right)^2+\left(x_2\right)^2\right]\right)\leqslant 1\,. \tag{50}$$

Actually, it is sufficient to find all the probabilities $q_{\text{opt}}(0,\,0)$, $q_{\text{opt}}(1,\,0)$ and $q_{\text{opt}}(0,\,1)$ which for any point (3) and $\forall\ \alpha\in\left(0;\,\frac{1}{2}\right]$ make the function

$$f\left(q_{\text{opt}}(0,\,0),\,q_{\text{opt}}(1,\,0),\,q_{\text{opt}}(0,\,1);\,x_1,\,x_2;\,\alpha\right)=$$
$$=q_{\text{opt}}(0,\,0)+q_{\text{opt}}(1,\,0)\exp\left[-\alpha\left(1-2x_1\right)\right]+q_{\text{opt}}(0,\,1)\exp\left[-\alpha\left(1-2x_2\right)\right]+$$
$$+q_{\text{opt}}(1,\,1)\exp\left[-\alpha\left(1-2x_1\right)-\alpha\left(1-2x_2\right)\right]-\exp\left(\alpha\left[\left(x_1\right)^2+\left(x_2\right)^2\right]-\frac{\alpha}{2}\right) \tag{51}$$

nonpositive, as it follows from (49).

Since the hypersurface (51) is a function of the six variables, then there is a very complicated analytical investigation of the nonpositive sign of this function, defined in the corresponding subspace of the six-dimensional arithmetical space $\mathbb{R}^6$. Firstly suppose, that there the two nonzero probabilities $q_{\text{opt}}(0,\,0)$ and $q_{\text{opt}}(1,\,1)$ in the vector (42), which certainly ought to be identical, recalling the symmetry conception in the being investigated game. For verifying whether the function

$$f\left(\frac{1}{2},\,0,\,0;\,x_1,\,x_2;\,\alpha\right)=\frac{1}{2}+\frac{1}{2}\exp\left[-\alpha\left(1-2x_1\right)-\alpha\left(1-2x_2\right)\right]-$$
$$-\exp\left(\alpha\left[\left(x_1\right)^2+\left(x_2\right)^2\right]-\frac{\alpha}{2}\right) \tag{52}$$

is nonpositive for any point (3) and $\forall\ \alpha\in\left(0;\,\frac{1}{2}\right]$ there has been constructed a general code script `VOMS` (figure 1) within the Technical Computing environment MATLAB, where the corresponding nonpositiveness of the hypersurface (51) may be numerically verified for any values of the vector (42).

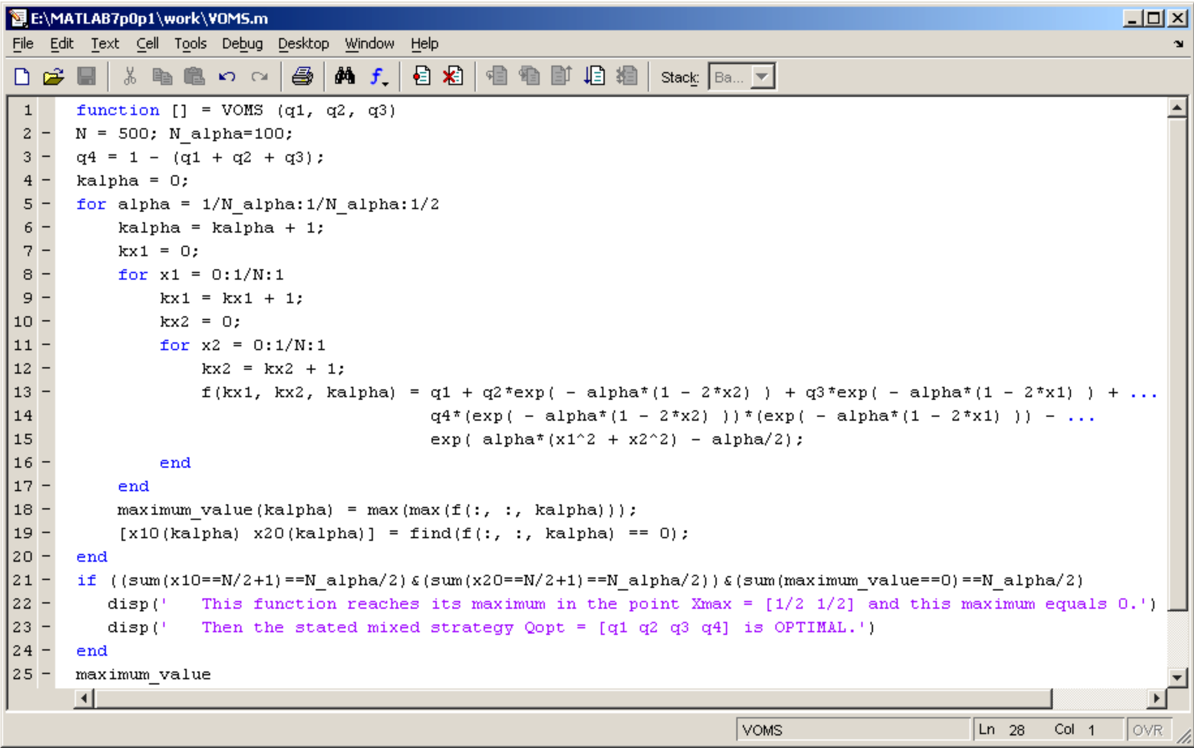


Figure 1. General code script VOMS for verifying the mixed strategy (42) for its optimality

The numerical verification for the nonpositiveness of the hypersurface (52) confirms, that the probabilities vector

$$\mathbf{Q}_{\text{opt}} = \begin{bmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{bmatrix} \tag{53}$$

is the optimal mixed strategy of the prey (figure 2).

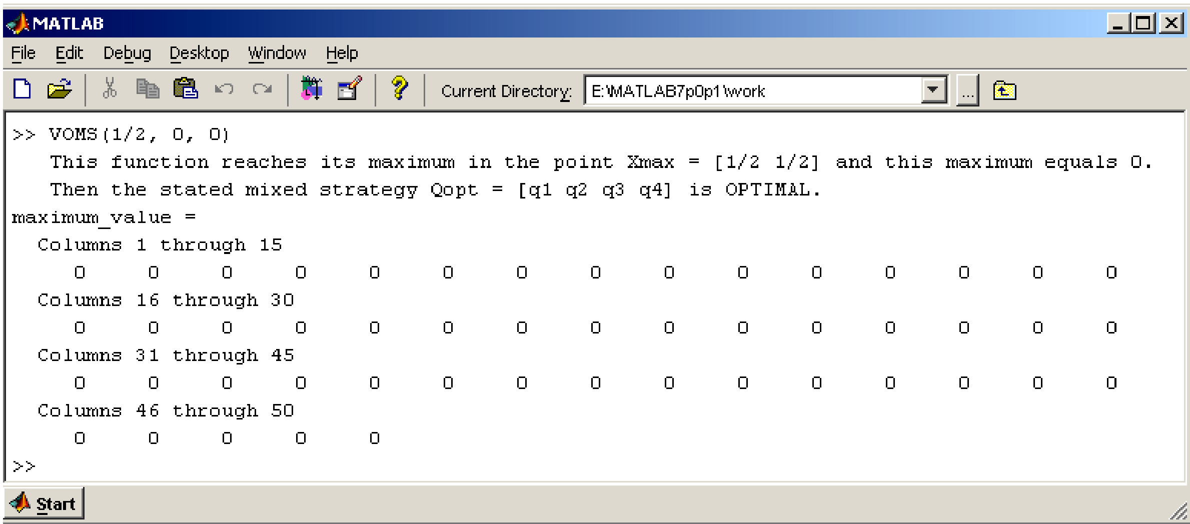


Figure 2. Result of the code VOMS execution for verifying the nonpositiveness of the hypersurface (52)

Using the symmetry conception, as the function

$$f\left(0, \frac{1}{2}, \frac{1}{2}; x_1, x_2; \alpha\right) = \frac{1}{2}\exp\left[-\alpha\left(1-2x_1\right)\right] + \frac{1}{2}\exp\left[-\alpha\left(1-2x_2\right)\right] -$$

$$-\exp\left(\alpha\left[\left(x_1\right)^2+\left(x_2\right)^2\right]-\frac{\alpha}{2}\right) \tag{54}$$

is nonpositive (figure 3) for any point (3) and $\forall\ \alpha \in \left(0;\ \frac{1}{2}\right]$, then the vector

$$\mathbf{Q}_{\mathrm{opt}}=\begin{bmatrix}0 & \frac{1}{2} & \frac{1}{2} & 0\end{bmatrix} \tag{55}$$

is another optimal mixed strategy of the prey.

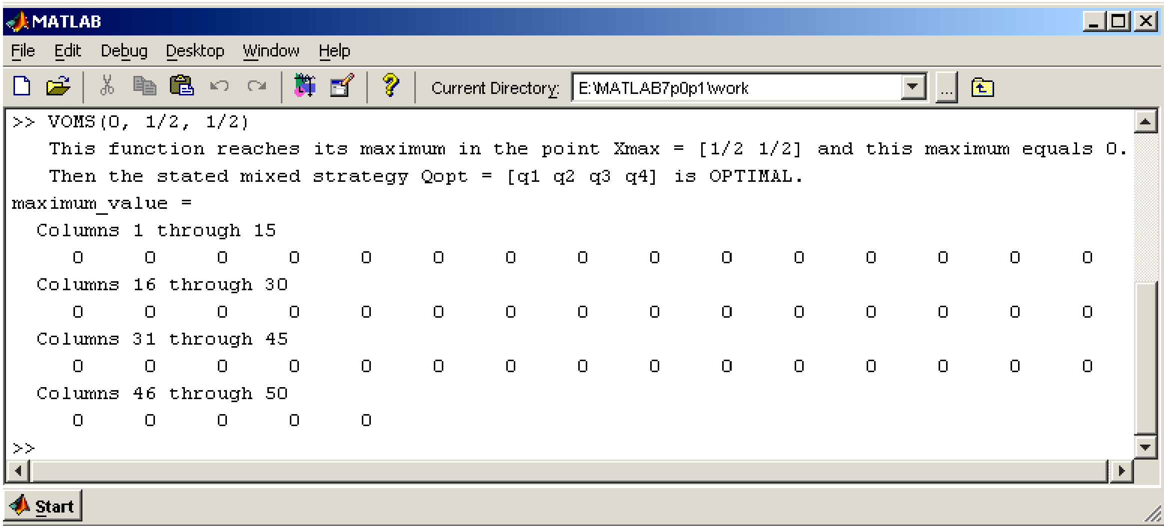


Figure 3. Result of the code VOMS execution for verifying the nonpositiveness of the hypersurface (54)

It is extraordinary that the vector

$$\mathbf{Q}_{\mathrm{opt}}=\begin{bmatrix}\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4}\end{bmatrix} \tag{56}$$

appears also to be the optimal mixed strategy of the prey, as the corresponding hypersurface

$$f\left(\frac{1}{4},\frac{1}{4},\frac{1}{4};\ x_1,\ x_2;\ \alpha\right)=\frac{1}{4}+\frac{1}{4}\exp\left[-\alpha\left(1-2x_1\right)\right]+\frac{1}{4}\exp\left[-\alpha\left(1-2x_2\right)\right]+$$
$$+\frac{1}{4}\exp\left[-\alpha\left(1-2x_1\right)-\alpha\left(1-2x_2\right)\right]-\exp\left(\alpha\left[\left(x_1\right)^2+\left(x_2\right)^2\right]-\frac{\alpha}{2}\right) \tag{57}$$

turns nonpositive (figure 4). The extraordinariness is in the break of the known theorem, saying that in the concave antagonistic game, where the first player has the pure strategies subset of the space $\mathbb{R}^m$, not lying in the space $\mathbb{R}^{m-1}$, the second player has the optimal strategy, mixed with no greater than with $m+1$ pure strategies of the second player, $m \in \mathbb{N}$ [2, p. 89].

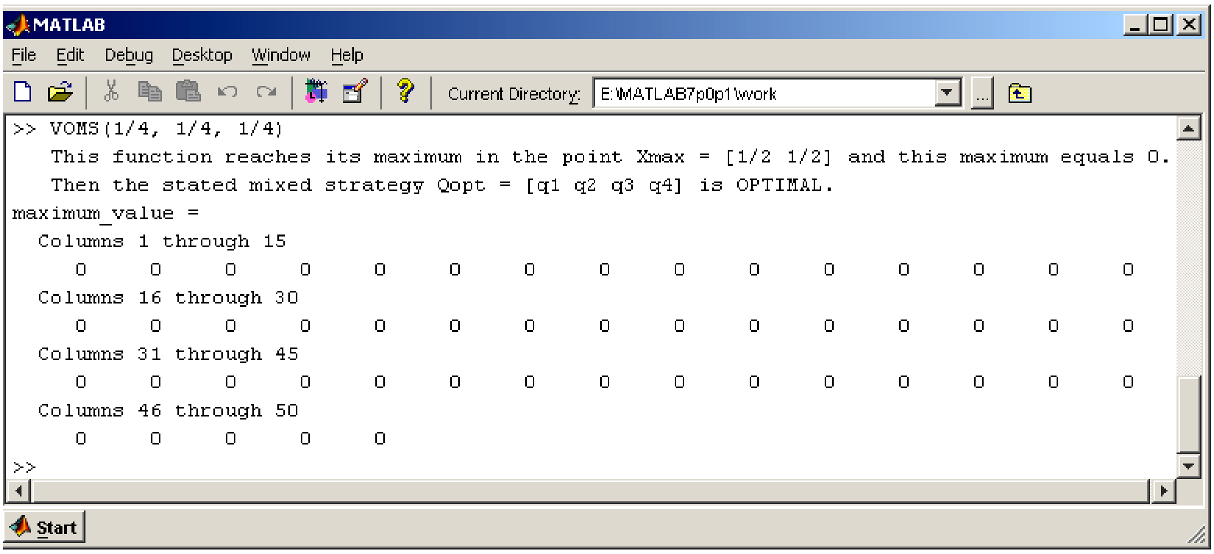


Figure 4. Result of the code VOMS execution, proving the nonpositiveness of the hypersurface (57)

The further symmetry conception application is in verifying the probabilities vectors

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \end{bmatrix}, \tag{58}$$

$$\begin{bmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}, \tag{59}$$

$$\begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{bmatrix}, \tag{60}$$

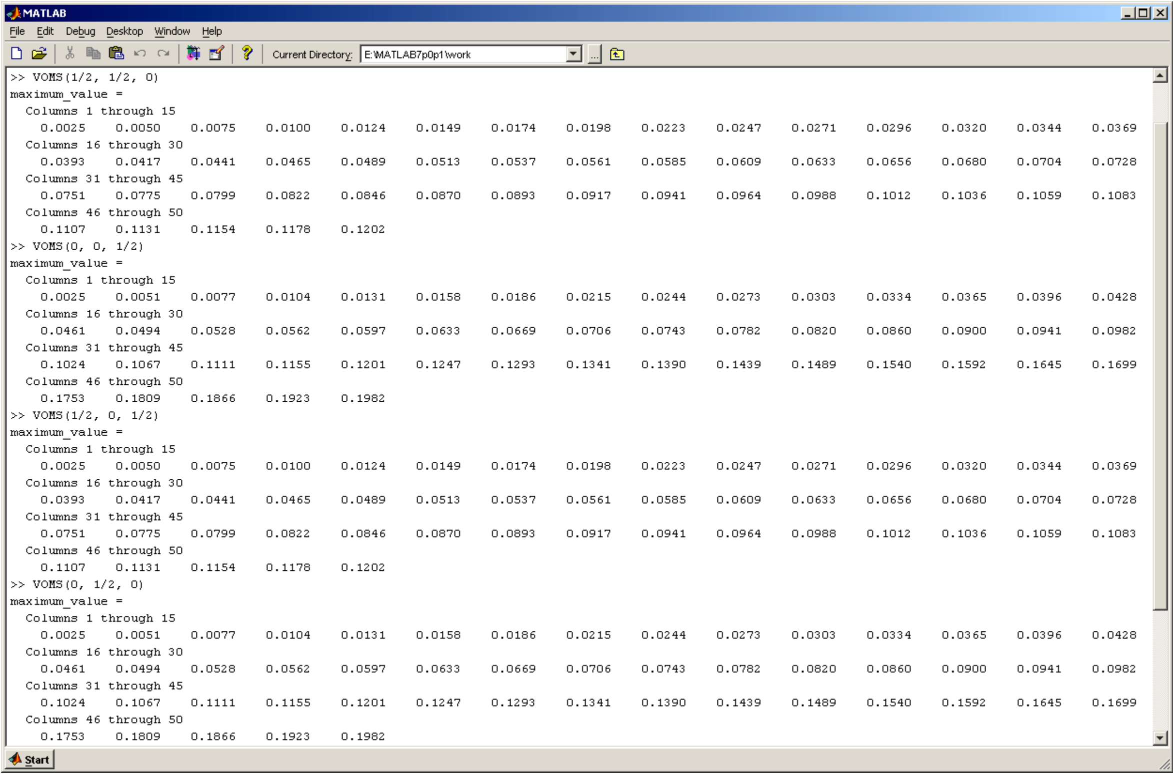
$$\begin{bmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \tag{61}$$

for their optimality. However, here the four corresponding functions (51) appear not to be necessarily nonpositive (figure 5), and so the probabilities vectors (58) — (61) are not the optimal mixed strategies of the prey.

Recalling back that theorem [2, p. 89] about the number of the pure strategies within the optimal mixture, here $m = 2$ and the probabilities vectors

$$\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{bmatrix}, \tag{62}$$

$$\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \end{bmatrix}, \tag{63}$$



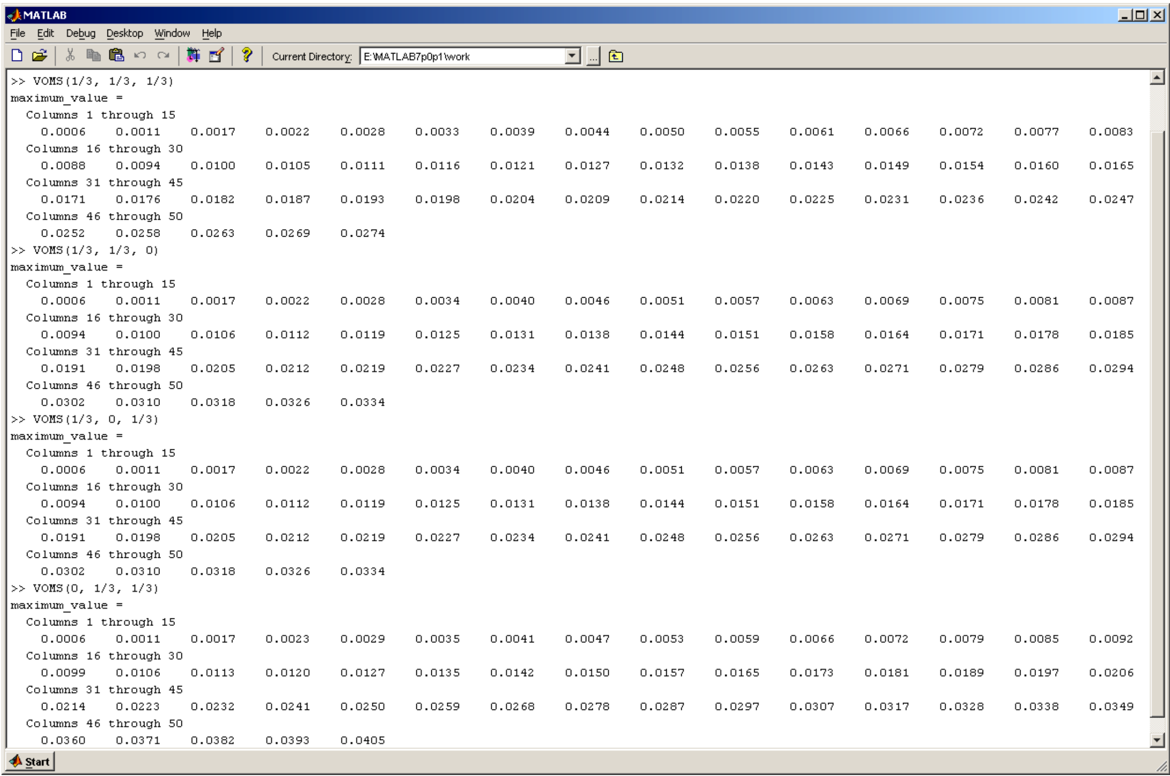
```
>> VOMS(1/2, 1/2, 0)
maximum_value =
  Columns 1 through 15
    0.0025    0.0050    0.0075    0.0100    0.0124    0.0149    0.0174    0.0198    0.0223    0.0247    0.0271    0.0296    0.0320    0.0344    0.0369
  Columns 16 through 30
    0.0393    0.0417    0.0441    0.0465    0.0489    0.0513    0.0537    0.0561    0.0585    0.0609    0.0633    0.0656    0.0680    0.0704    0.0728
  Columns 31 through 45
    0.0751    0.0775    0.0799    0.0822    0.0846    0.0870    0.0893    0.0917    0.0941    0.0964    0.0988    0.1012    0.1036    0.1059    0.1083
  Columns 46 through 50
    0.1107    0.1131    0.1154    0.1178    0.1202
>> VOMS(0, 0, 1/2)
maximum_value =
  Columns 1 through 15
    0.0025    0.0051    0.0077    0.0104    0.0131    0.0158    0.0186    0.0215    0.0244    0.0273    0.0303    0.0334    0.0365    0.0396    0.0428
  Columns 16 through 30
    0.0461    0.0494    0.0528    0.0562    0.0597    0.0633    0.0669    0.0706    0.0743    0.0782    0.0820    0.0860    0.0900    0.0941    0.0982
  Columns 31 through 45
    0.1024    0.1067    0.1111    0.1155    0.1201    0.1247    0.1293    0.1341    0.1390    0.1439    0.1489    0.1540    0.1592    0.1645    0.1699
  Columns 46 through 50
    0.1753    0.1809    0.1866    0.1923    0.1982
>> VOMS(1/2, 0, 1/2)
maximum_value =
  Columns 1 through 15
    0.0025    0.0050    0.0075    0.0100    0.0124    0.0149    0.0174    0.0198    0.0223    0.0247    0.0271    0.0296    0.0320    0.0344    0.0369
  Columns 16 through 30
    0.0393    0.0417    0.0441    0.0465    0.0489    0.0513    0.0537    0.0561    0.0585    0.0609    0.0633    0.0656    0.0680    0.0704    0.0728
  Columns 31 through 45
    0.0751    0.0775    0.0799    0.0822    0.0846    0.0870    0.0893    0.0917    0.0941    0.0964    0.0988    0.1012    0.1036    0.1059    0.1083
  Columns 46 through 50
    0.1107    0.1131    0.1154    0.1178    0.1202
>> VOMS(0, 1/2, 0)
maximum_value =
  Columns 1 through 15
    0.0025    0.0051    0.0077    0.0104    0.0131    0.0158    0.0186    0.0215    0.0244    0.0273    0.0303    0.0334    0.0365    0.0396    0.0428
  Columns 16 through 30
    0.0461    0.0494    0.0528    0.0562    0.0597    0.0633    0.0669    0.0706    0.0743    0.0782    0.0820    0.0860    0.0900    0.0941    0.0982
  Columns 31 through 45
    0.1024    0.1067    0.1111    0.1155    0.1201    0.1247    0.1293    0.1341    0.1390    0.1439    0.1489    0.1540    0.1592    0.1645    0.1699
  Columns 46 through 50
    0.1753    0.1809    0.1866    0.1923    0.1982
```

Figure 5. The code VOMS execution as the proof of the nonoptimality of (58) — (61)

$$\begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{bmatrix}, \tag{64}$$

$$\begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \tag{65}$$

must be verified for their optimality. But analogously there is the visualized proof (figure 6), that any of the probabilities vectors (62) — (65) is not the optimal mixed strategy of the prey.

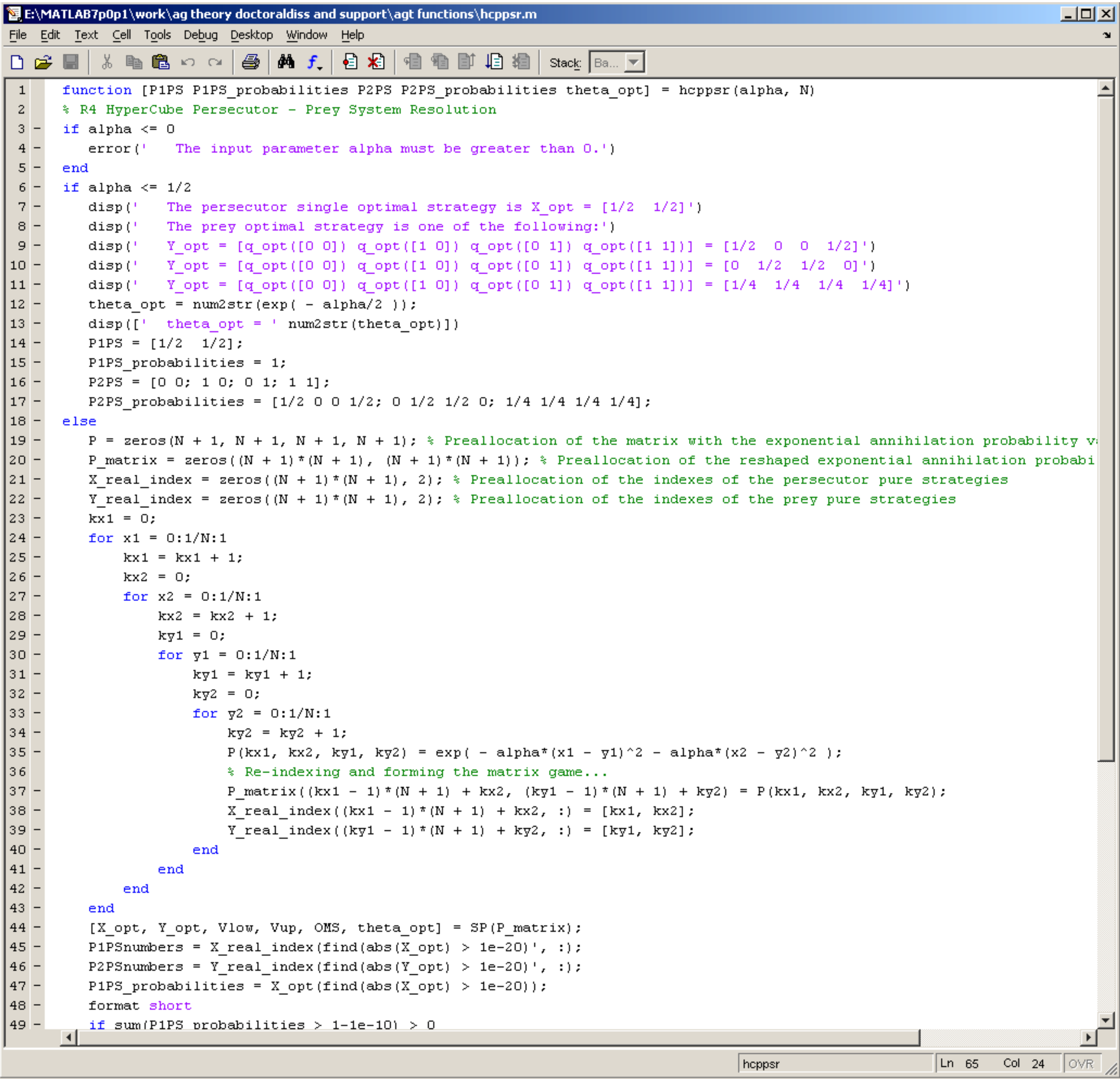


```
>> VOMS(1/3, 1/3, 1/3)
maximum_value =
  Columns 1 through 15
    0.0006    0.0011    0.0017    0.0022    0.0028    0.0033    0.0039    0.0044    0.0050    0.0055    0.0061    0.0066    0.0072    0.0077    0.0083
  Columns 16 through 30
    0.0088    0.0094    0.0100    0.0105    0.0111    0.0116    0.0121    0.0127    0.0132    0.0138    0.0143    0.0149    0.0154    0.0160    0.0165
  Columns 31 through 45
    0.0171    0.0176    0.0182    0.0187    0.0193    0.0198    0.0204    0.0209    0.0214    0.0220    0.0225    0.0231    0.0236    0.0242    0.0247
  Columns 46 through 50
    0.0252    0.0258    0.0263    0.0269    0.0274
>> VOMS(1/3, 1/3, 0)
maximum_value =
  Columns 1 through 15
    0.0006    0.0011    0.0017    0.0022    0.0028    0.0034    0.0040    0.0046    0.0051    0.0057    0.0063    0.0069    0.0075    0.0081    0.0087
  Columns 16 through 30
    0.0094    0.0100    0.0106    0.0112    0.0119    0.0125    0.0131    0.0138    0.0144    0.0151    0.0158    0.0164    0.0171    0.0178    0.0185
  Columns 31 through 45
    0.0191    0.0198    0.0205    0.0212    0.0219    0.0227    0.0234    0.0241    0.0248    0.0256    0.0263    0.0271    0.0279    0.0286    0.0294
  Columns 46 through 50
    0.0302    0.0310    0.0318    0.0326    0.0334
>> VOMS(1/3, 0, 1/3)
maximum_value =
  Columns 1 through 15
    0.0006    0.0011    0.0017    0.0022    0.0028    0.0034    0.0040    0.0046    0.0051    0.0057    0.0063    0.0069    0.0075    0.0081    0.0087
  Columns 16 through 30
    0.0094    0.0100    0.0106    0.0112    0.0119    0.0125    0.0131    0.0138    0.0144    0.0151    0.0158    0.0164    0.0171    0.0178    0.0185
  Columns 31 through 45
    0.0191    0.0198    0.0205    0.0212    0.0219    0.0227    0.0234    0.0241    0.0248    0.0256    0.0263    0.0271    0.0279    0.0286    0.0294
  Columns 46 through 50
    0.0302    0.0310    0.0318    0.0326    0.0334
>> VOMS(0, 1/3, 1/3)
maximum_value =
  Columns 1 through 15
    0.0006    0.0011    0.0017    0.0023    0.0029    0.0035    0.0041    0.0047    0.0053    0.0059    0.0066    0.0072    0.0079    0.0085    0.0092
  Columns 16 through 30
    0.0099    0.0106    0.0113    0.0120    0.0127    0.0135    0.0142    0.0150    0.0157    0.0165    0.0173    0.0181    0.0189    0.0197    0.0206
  Columns 31 through 45
    0.0214    0.0223    0.0232    0.0241    0.0250    0.0259    0.0268    0.0278    0.0287    0.0297    0.0307    0.0317    0.0328    0.0338    0.0349
  Columns 46 through 50
    0.0360    0.0371    0.0382    0.0393    0.0405
```

Figure 6. The code VOMS execution as the proof of the nonoptimality of (62) — (65)

Consequently, $\forall\ \alpha \in \left(0;\ \frac{1}{2}\right]$ there is the analytical solution sufficiency of the stated concave game on the unit hypercube (6) with the kernel (5), where the persecutor has the generally single optimal strategy (34), and the prey should apply one of its optimal mixed strategies, expressed with the optimal probabilities vectors (53), (55), (56). This gives the optimal probability (33) of the prey annihilation.

Further will take the parameter $\alpha > \frac{1}{2}$ as the quality of the old or low-technology equipment for the persecutor. Though the exact analytic methods of solving such system persecutor — prey configuration are unavailable, there may be used the MATLAB software for numerical computations. On the figure 7 there is visualized the MATLAB program module `hcppsr` (Hypercube Persecutor — Prey System Resolution) code for solving the antagonistic game with the kernel (5), defined on the unit hypercube (6) for any $\alpha > 0$.



```
E:\MATLAB7p0p1\work\ag theory doctoraldiss and support\agt functions\hcppsr.m

function [P1PS P1PS_probabilities P2PS P2PS_probabilities theta_opt] = hcppsr(alpha, N)
% R4 HyperCube Persecutor - Prey System Resolution
if alpha <= 0
   error('   The input parameter alpha must be greater than 0.')
end
if alpha <= 1/2
   disp('   The persecutor single optimal strategy is X_opt = [1/2  1/2]')
   disp('   The prey optimal strategy is one of the following:')
   disp('   Y_opt = [q_opt([0 0]) q_opt([1 0]) q_opt([0 1]) q_opt([1 1])] = [1/2  0  0  1/2]')
   disp('   Y_opt = [q_opt([0 0]) q_opt([1 0]) q_opt([0 1]) q_opt([1 1])] = [0  1/2  1/2  0]')
   disp('   Y_opt = [q_opt([0 0]) q_opt([1 0]) q_opt([0 1]) q_opt([1 1])] = [1/4  1/4  1/4  1/4]')
   theta_opt = num2str(exp( - alpha/2 ));
   disp(['  theta_opt = ' num2str(theta_opt)])
   P1PS = [1/2  1/2];
   P1PS_probabilities = 1;
   P2PS = [0 0; 1 0; 0 1; 1 1];
   P2PS_probabilities = [1/2 0 0 1/2; 0 1/2 1/2 0; 1/4 1/4 1/4 1/4];
else
   P = zeros(N + 1, N + 1, N + 1, N + 1); % Preallocation of the matrix with the exponential annihilation probability v
   P_matrix = zeros((N + 1)*(N + 1), (N + 1)*(N + 1)); % Preallocation of the reshaped exponential annihilation probabi
   X_real_index = zeros((N + 1)*(N + 1), 2); % Preallocation of the indexes of the persecutor pure strategies
   Y_real_index = zeros((N + 1)*(N + 1), 2); % Preallocation of the indexes of the prey pure strategies
   kx1 = 0;
   for x1 = 0:1/N:1
       kx1 = kx1 + 1;
       kx2 = 0;
       for x2 = 0:1/N:1
           kx2 = kx2 + 1;
           ky1 = 0;
           for y1 = 0:1/N:1
               ky1 = ky1 + 1;
               ky2 = 0;
               for y2 = 0:1/N:1
                   ky2 = ky2 + 1;
                   P(kx1, kx2, ky1, ky2) = exp( - alpha*(x1 - y1)^2 - alpha*(x2 - y2)^2 );
                   % Re-indexing and forming the matrix game...
                   P_matrix((kx1 - 1)*(N + 1) + kx2, (ky1 - 1)*(N + 1) + ky2) = P(kx1, kx2, ky1, ky2);
                   X_real_index((kx1 - 1)*(N + 1) + kx2, :) = [kx1, kx2];
                   Y_real_index((ky1 - 1)*(N + 1) + ky2, :) = [ky1, ky2];
               end
           end
       end
   end
   [X_opt, Y_opt, Vlow, Vup, OMS, theta_opt] = SP(P_matrix);
   P1PSnumbers = X_real_index(find(abs(X_opt) > 1e-20)', :);
   P2PSnumbers = Y_real_index(find(abs(Y_opt) > 1e-20)', :);
   P1PS_probabilities = X_opt(find(abs(X_opt) > 1e-20));
   format short
   if sum(P1PS_probabilities > 1-1e-10) > 0

hcppsr    Ln 65    Col 24    OVR
```

Figure 7. MATLAB program module `hcppsr` code for solving the antagonistic game with the kernel (5), defined on the unit hypercube (6) for any $\alpha > 0$

The embodied module handles the two inputs: the parameter α and the number of the points of each segment $[0;\ 1]$ for the numerical presentation of the kernel (5). The greater this number the longer the computation of the equilibrium situation in the system, though the accuracy increases. Some of the system resolutions by the fixed α are exampled on the figures 8 — 11, where it is clear, that the number of pure strategies of the persecutor and prey, selected with nonzero optimal probabilities, is increasing on average with the greater α.

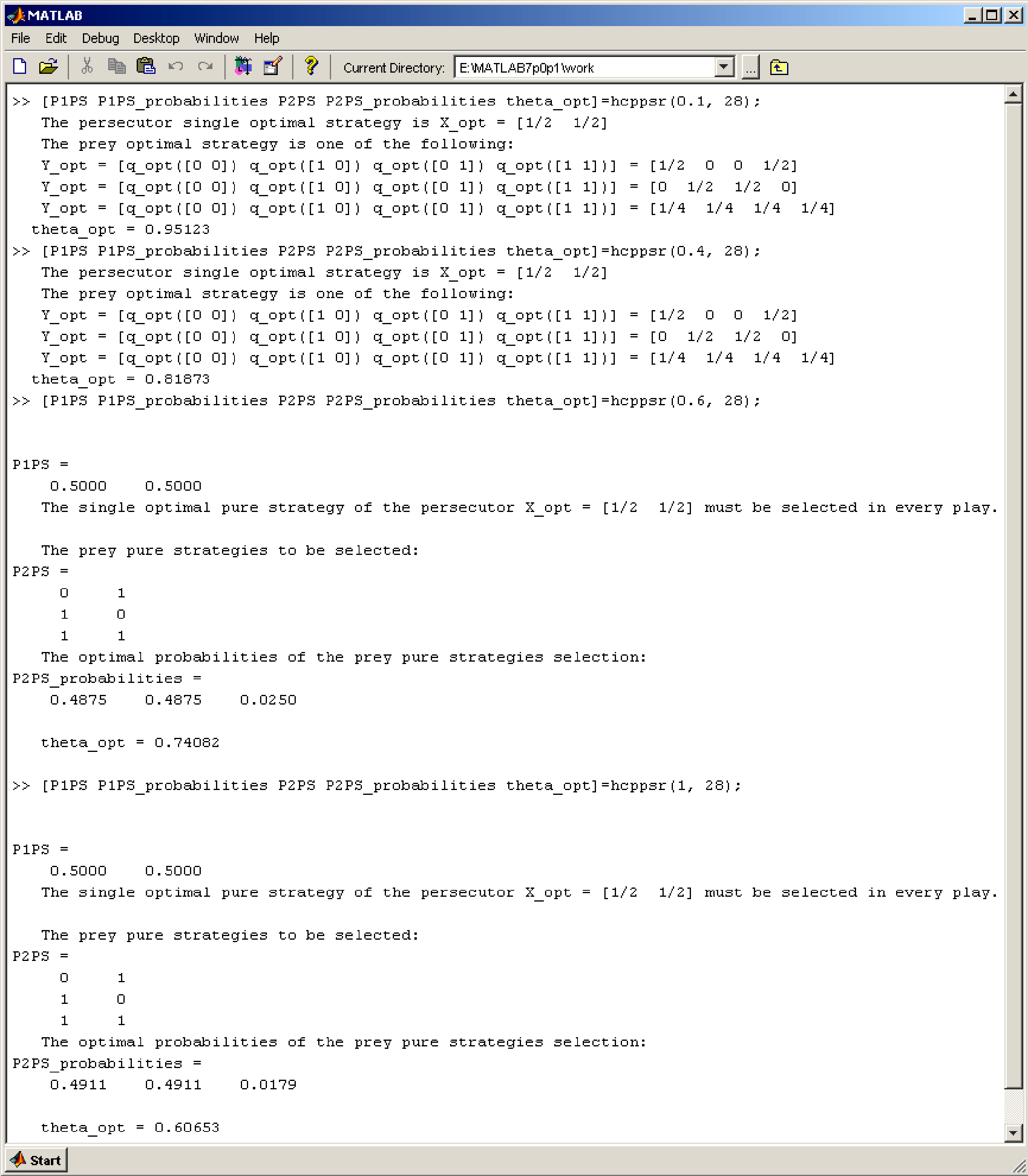


Figure 8. Solutions for $\alpha \in \{0.1,\ 0.4,\ 0.6,\ 1\}$ and 28 breaking points of each segment $[0;\ 1]$

The figures 9 — 11 have been print-screened by the 20 breaking points of each segment $[0;\ 1]$, though this factor decreased the accuracy. Nevertheless, with the powerful computing systems there is the real might to increase the accuracy as needed, taking the greater number of the unit segment breaking points. Thus the non-equiprobable pure strategies of the prey on the figure 9 may be explained with the finite precision of calculations, being fulfilled within the program submodule SP, accepting the kernel in the matrix form and returning the game solution [7 — 9].

However, after the figure 11, it should be marked, that the persecutor — prey systems with the optimal probability $\theta_{\text{opt}} < \frac{1}{2}$ of the prey strike are out of practical interest. Then the figure 8 displays nearly the boundary worst system configuration resolution, where the persecutor still has the single optimal pure strategy (34). Speaking generally, $\forall\ \alpha \in (0;\ 2]$ the persecutor single optimal pure strategy (34) holds still. For the more accurate calculation of the prey optimal behavior there certainly should be increased the number of the unit segment breaking points.

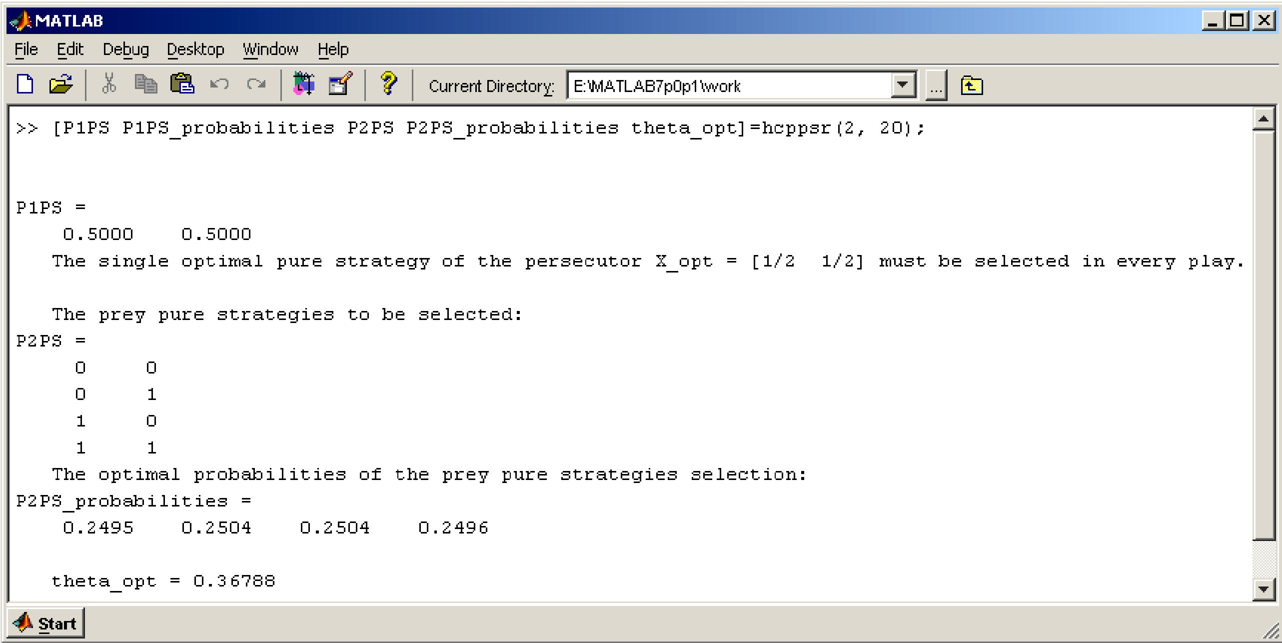


Figure 9. Solution for $\alpha = 2$ and 20 breaking points of each segment $[0;\ 1]$

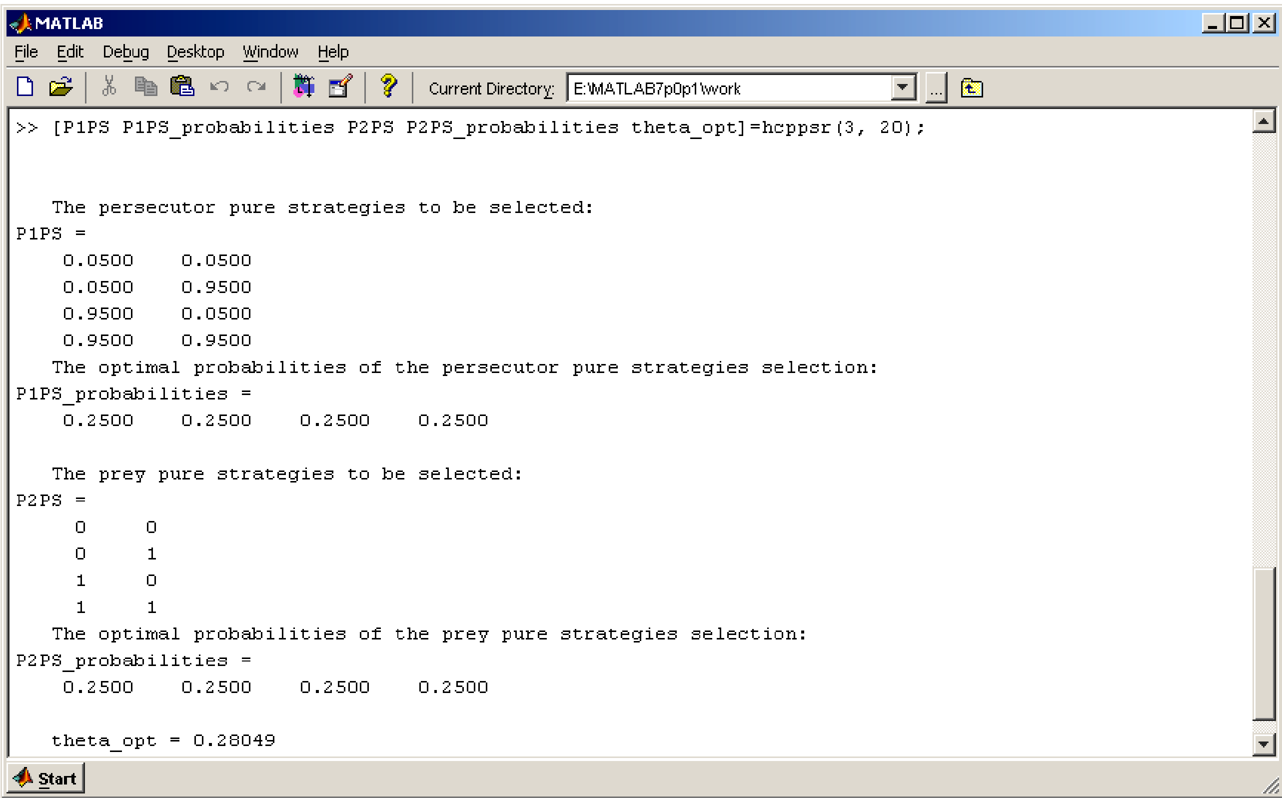


Figure 10. Solution for $\alpha = 3$ and 20 breaking points of each segment $[0;\ 1]$

Conclusion. The resolution way of the persecutor — prey system with the kernel (5) definitely depends on the positive parameter α. For $\alpha \in \left(0; \frac{1}{2}\right]$ the solution is the single optimal strategy of the persecutor (34) and one of the optimal mixed strategies (53), (55), (56) of the prey; at that the game value is $\theta_{opt} = \exp\left(-\frac{\alpha}{2}\right)$. For $\alpha > \frac{1}{2}$ there should be applied the constructed MATLAB program module hcppsr, returning the approximate solution [10 — 21] as the result of the corresponding numerical computation. This approximate solution accuracy may be increased as needed, though on protracting the resolution process.

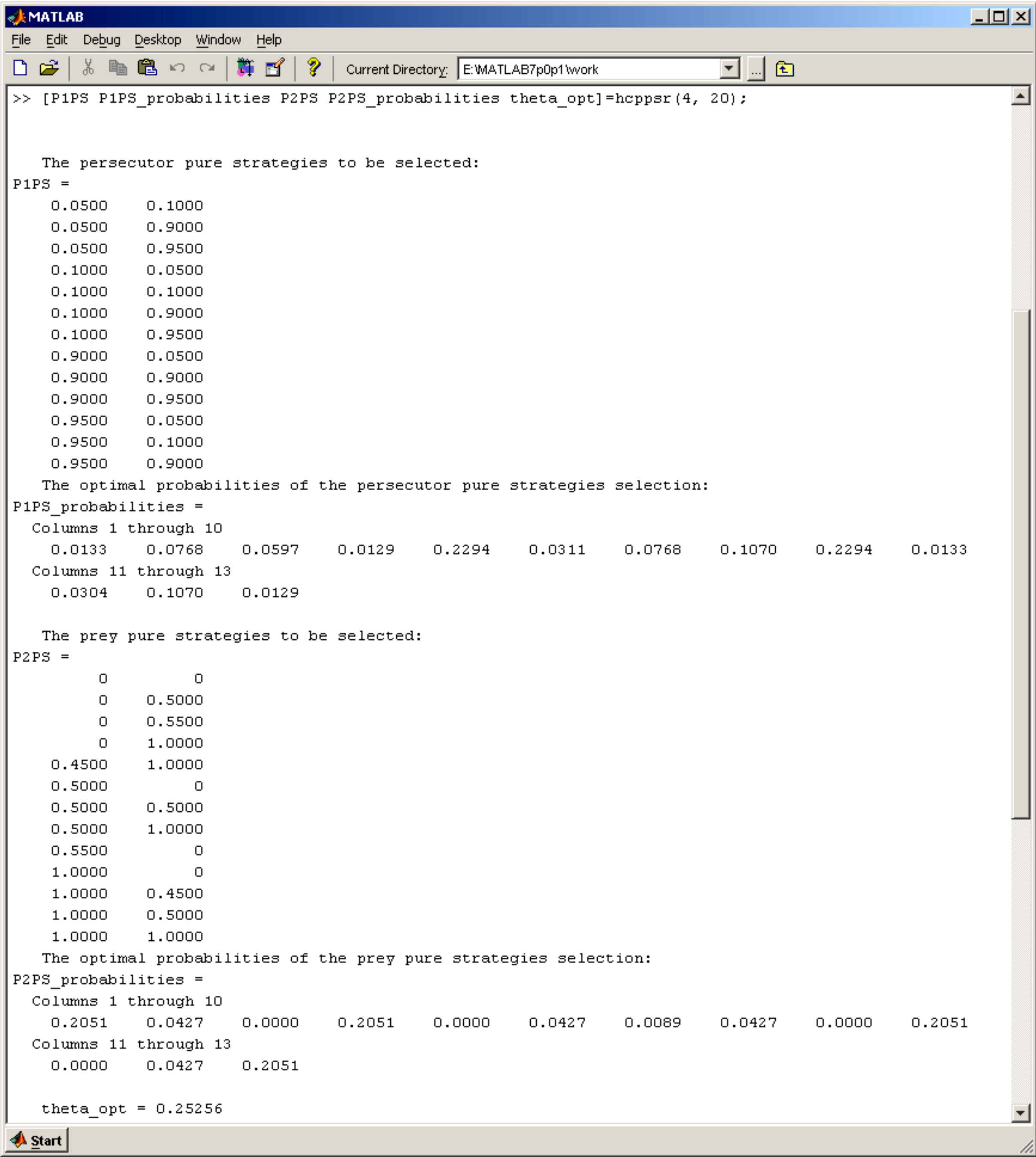


Figure 11. Solution for $\alpha = 4$ and 20 breaking points of each segment $[0; 1]$

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